1. ACCELERATOR PHYSICS OF COLLIDERS

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The event rate R in a collider is proportional to the interaction cross section $\sigma_{\rm int}$ and the factor of proportionality is called the *luminosity*:

$$R = \mathscr{L}\sigma_{\rm int} \ .$$

If two bunches containing n_1 and n_2 particles collide with frequency f, the luminosity is

$$\mathscr{L} = f\frac{n_1 n_2}{4\pi\sigma_x\sigma_y} \tag{1.2}$$

where σ_x and σ_y characterize the Gaussian transverse beam profiles in the horizontal (bend) and vertical directions and to simplify the expression it is assumed that the bunches are identical in transverse profile, that the profiles are independent of position along the bunch, and the particle distributions are not altered during collision. Whatever the distribution at the source, by the time the beam reaches high energy, the normal form is a good approximation thanks to the central limit theorem of probability and the diminished importance of space charge effects.

The beam size can be expressed in terms of two quantities, one termed the *transverse emittance*, ϵ, and the other, the *amplitude function*, β. The transverse emittance is a beam quality concept reflecting the process of bunch preparation, extending all the way back to the source for hadrons and, in the case of electrons, mostly dependent on synchrotron radiation. The amplitude function is a beam optics quantity and is determined by the accelerator magnet configuration. When expressed in terms of σ and β the transverse emittance becomes

$$\epsilon = \pi\sigma^2/\beta \ .$$

Of particular significance is the value of the amplitude function at the interaction point, β^*. Clearly one wants β^* to be as small as possible; how small depends on the capability of the hardware to make a near-focus at the interaction point.

Eq. (1.2) can now be recast in terms of emittances and amplitude functions as

$$\mathscr{L} = f\frac{n_1 n_2}{4\sqrt{\epsilon_x\,\beta_x^*\,\epsilon_y\,\beta_y^*}} \ . \tag{1.10}$$

Thus, to achieve high luminosity, all one has to do is make high population bunches of low emittance to collide at high frequency at locations where the beam optics provides as low values of the amplitude functions as possible.

Further discussion and references may be found in the full *Review of Particle Physics.*